C-Theory and the Epistemology of Mathematics: Explanations and Time Symmetry

Strahinja Đorđević

Abstract: The objective of this paper is to investigate the possibility of a conditional symmetry in temporal ordering which pertains to the epistemology of mathematics. In short, I will try to provide the groundwork for analyzing one form of symmetrical temporality (from the perspective of an epistemic agent) by embracing the C-theory and non-causal time reversal. It will be argued that, in addition to the general account of time, which is tightly aligned with temporal ordering and seemingly asymmetric, there could possibly exist another form of temporality which is manifested in the so-called *epistemic temporal symmetry*. This symmetry relies only on the C-series, does not violate causal symmetry, and applies exclusively to the mathematical epistemic domain. In order to adequately describe epistemic time symmetry in mathematics, I will present some examples of mathematical explanations in which this supposed symmetry can be found.

Keywords: Time Symmetry; Mathematical Explanations; C-theory; Time Reversal; Causation.

1. Temporal Overtone of Mathematical Explanations Outside of Pure Mathematics

Our idea is to argue two important things, (1) there is a kind of *explanatory symmetry* in mathematics, (2) this symmetry can imply *time symmetry* (temporal-bidirectionality) in a specific epistemic sense which is enabled by the *C-theory* of time. The first thing we need to emphasize is that there is a generally widespread acceptance of the view that pure mathematics and its objects are not located in time (nor in space), so, strictly speaking, there is no temporality in the mathematical domain. But one important thing that mustn't slip our minds is that when mathematics 'crosses' into the empirical domain — we have to come in contact with some form of temporality. In other words, mathematics and its entities are considered objects that are not in space and time, however, in our application of mathematics, there is always a certain temporal sequence. With this in mind, we will argue here that there is a possibility of temporal symmetry from the perspective of an epistemic agent involved in mathematical procedures, in an implicit sense, which means that (there are cases in which) it doesn't matter which part of the mathematical explanations we consider first in the time-directional sense. This means that symmetry in mathematical explanations, which we will analyze in later sections, implies a certain kind of symmetric temporality, albeit only from an epistemic or even pre-epistemic perspective.

To speak more succinctly, the main issue I want to address in this research is that which to a large extent corresponds to the question: Is it possible to speak of something more than regular asymmetric temporality in mathematical explanations? I consider the answer to be resoundingly affirmative. Below, I will start from the thesis that time in mathematical explanations has no explicit direction and flow, considering the fact that the domain in which the epistemic agent is in, although not mathematical, is not purely empirical either. Hence, unlike in pure mathematical domain, time does exist, and dissimilar to the empirical world, time does not have the same characteristics (be it direction, flow, or anything else). With all of this in mind, I will try to defend the notion of *epistemic temporal symmetry*.

2. Introducing Explanatory Symmetry

When we consider the examples of symmetry that occurs in mathematical explanations, it is most gratifying to refer to the case of the so-called Zeitz's[1] biased coin.[2] This kind of 'experiment' takes into consideration a 'biased' coin, which has a predetermined probability p, which refers to the head of the coin. When throwing such a coin, it could be concluded that the probability for each head fall number from 0 to n is the same, that is, $1 / n + 1$. So, if we, like Zeitz, take an example where the number of throws is 2000, it would soon become obvious that we have 2001 possible numbers of heads falling (between 0 and 2000). The predetermination value ranges from 0 to 1 and is randomly determined. This means that values like 0.138 will denote a smaller chance of heads, and values like 0.722 will indicate that the chance for heads is bigger. However, the results suggest that the chance of getting any number p of heads, be that 1000 (that intuitively acts as perhaps the most solid option) or any other randomly chosen number that would, to a greater or lesser extent, be closer to zero or one, is the same.[3] "The amazing answer is that the probability [of 1000 heads in 2000 tosses] is 1/ 2001. Indeed, it doesn't matter how many heads we wish to see — for any [integer r] between 0 and 2000, the probability that r heads occur [in 2000 tosses] is 1/ 2001" (Zeitz 2000: 2). The same will apply to every other number of throws (200 throws / 201 possible head falls numbers, 4000 throws / 4001 possible head falls number), no matter

Filozofski fakultet, Univerzitet u Beogradu
Belgrade, Serbia
email: boostergold@hotmail.rs

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what value between 0 and 1 will emerge, the probability will be 1 / n + 1.

What stands out from the example of Zeitz's coin is that there are two different kinds of proof for why the probability is always 1 / n + 1.[4] Both proofs are correct, however, the first proof is different from the second in that it gives us no explanation as to why for each number of throws n the result is always the same. The second proof, on the other hand, gives us an explanation as to why symmetry in the result (with respect to the setting) manifests itself, and that symmetry is also the reason why the example with the Zeitz's coin is so important. As much as one is versed in mathematics, whether layman or expert, the fact is that such examples will attract more attention than some other mathematical settings. This is also noticed by Lange when he states that salience is one of the main criteria that underpin our 'inquisitive thirst' for explanations: "The symmetry, once having become salient, prompts the demand for an explanation: a proof that traces the result back to a similar symmetry in the problem. In light of the symmetry's salience, there is a point in asking for an explanation over and above a proof" (Lange 2017: 239). Symmetry, perhaps more than anything else, makes certain aspects of the proof more salient, as is evident from this example.

How can we ultimately understand the implications of Zeitz's coin and what do they say about the symmetry of mathematics? First of all, this example is intended (from Lange's perspective) to shows us that there is a certain symmetry between the results of some mathematical proof and the procedure that led to it. Of course, this does not in itself imply any conclusion regarding its temporality, since the procedures themselves are nor temporal. However, as soon as an explanatory symmetric mathematical scenario is placed in the context of some epistemic agent's knowledge, then we can speak of a completely different kind of symmetry. The symmetry here is therefore twofold, we have symmetry between procedure and result which is the cause of the salience of some mathematical proof, and we have symmetrical epistemic relation (which may or may not imply temporality that differs from the standard view) between the procedure and the result which is the courtesy of the transition to the epistemic domain. This second type of symmetry allows for a non-standard interpretation of the relationship between setting and results. Therefore, we believe that it is a very important aspect of some mathematical proofs.

When it comes to symmetry in the case of mathematics, it is clear that it is a mathematical necessity manifested in (at least some) mathematical proofs and that this kind of salience requires further explanation. However, another big problem, in the author's opinion, should address the question as to whether mathematical symmetry can have an epistemico-temporal connotation. And by this, we mean a comprehensive account of the way our mathematical knowledge is formed in regards to how it is temporally organized. For almost all proponents of the existence of time, it is generally accepted that it is asymmetrical in nature, in other words, that there is only one time direction. The idea of symmetrical time directionality, at first glance, seems rather absurd, much more absurd than, say, the rather criticized notion of retro-causality (Fitzgerald 1974; van Putten 2006). But things may seem different if we consider the pure epistemic connotations and if our focus is not strictly aimed at the intuitive appeal concerning time directionality.

It could be argued that the directionality of time from the perspective of epistemic agents could potentially determine what a mathematical procedure is and what a mathematical result is. In some cases, this distinction really seems arbitrary, since something that was originally classified as a procedure can also be classified as a result and something that what was originally classified as a result can be also be classified as a procedure. In this way, the traditional boundary between cause and effect, as we will see, is not erased (*ergo* we will not claim that causal processes are symmetrical), but rather it is shown that mathematical operations could be analyzed (as we shall see, in a strictly epistemic sense) from two temporal directions, depending on the perspective from which it is 'epistemically approached'. In order to better understand this symmetry, we will present a theoretic strategy embracing the most suitable temporal theory for the description of the given phenomenon.

3. C-theory and Time Reversal Strategy

In the standard philosophy of time, one of the two most influential notions are the A-series and the B-series, introduced by John McTaggart. The A-series is a "series of positions which runs from the far past through the near past to the present, and then from the present through the near future to the far future, or conversely", and the B-series represents a "series of positions which runs from earlier to later, or conversely" (McTaggart: 458). In the broadest possible sense, the proponents of the former can be considered A-theorists, while the followers of the latter are called B-theorists.[5] To oversimplify this distinction, according to the A-theory time passes, on the other hand, the B-theory holds that time is an illusion (Prosser 2012: 92). This distinction is conditioned by the fact that the A-series holds that time has both direction and flow, of which the latter explains our experience of change and the B-series holds that time has only the direction. However, what if time has neither direction nor flow, the question that arises from this assumption is what would happen if we establish that both the A-theory and the B-theory are not sustainable. Should this view automatically be characterized as temporally nihilistic, or is there a third option?

It seems that such a solution exists. Until recently, there was very little discussion about the third series of positions — the C-series which is described as "not temporal, for it involves no change, but only an order. Events have an order. They are, let us say, in the order M, N, O, P. And they are therefore not in the order M, O, N, P, or O, N, M, P, or in any other possible order" (McTaggart: 462). Most proponents of both the A-theory and the B-theory, much like McTaggart, when he presents his arguments for the unreality of time, don't take into account the impact of the C-series to be relevant, since it, as originally defined, seems rather insufficient to be considered on its own. However, this should not be understood as a valid reason to neglect it. McTaggart remarked cautiously that the C-series can potentially 'survive' on its own, but that this, on the other hand, begs the question of whether it is a time series at all. The problem is that this is "a series which is not temporal has no direction of its own, though it has an order" (McTaggart: 462). But if the supposed scenario from the sections above holds, if time actually doesn't have any direction nor flow, then the only problem that remains is that the C-series is not temporal. But, thanks to recent insights from Matt Farr, who argues that the so-called C-theory is more preferable than both the A-theory and the B-theory, we saw that the C-series could be seen as temporal. He presents an interesting *time reversal* strategy, that refers to physical theories, but which we will in this research apply to mathematical explanations. In order to fully embrace this position, we have to deal with what is seemingly the biggest problem in acceptance of the C-theory — its alleged connection to causal symmetry acceptance. Alternatively stated, since we are defending the notion of symmetry of time, this naturally raises the question of symmetrical causation of mathematical explanations.

Causation is mostly seen as asymmetric, and this is not only because the cause seemingly temporally precedes the effect. There are many different asymmetrical features that can't be reduced to time order, such as agency or manipulability, counterfactual dependence, overdetermination, robustness, and more (Hausman 1998). Asymmetrical causation seems like a concept that is hard to refute. It also might seem that the time symmetry which derives from the acceptance of the C-theory implies causal symmetry, or in other words that causes and effects could take each other's roles, which is, to say the least, problematic. It indeed is hard to defend that the theory of temporal symmetry is consistent with the idea that cause precedes the effect and the effect temporally going after the cause, which might act as a threat to the C-theory. But, lately, it has been argued that time symmetry doesn't imply causal symmetry, which means that time reversal does not invert causal relations, which consequently enables our main thesis. According to this view, there are two main views regarding the relationship between time symmetry and causal symmetry (Farr 2020: 182):

Causal Time Reversal (CTR): Time reversal involves inverting causal relations, taking causes to effects and vice versa.

Non-causal Time Reversal (¬CTR): Time reversal does not invert causal relations; the distinction between cause and effect remains invariant under time reversal.

Just as the mathematical domain does not have to be temporal nor causal for the mathematical truths to reveal themselves to the epistemic subject in a temporal manner — time can be reversed without inverting causal relations, at least according to (¬CTR), which I defend. Matt Farr notes, there is "a causal and explanatory asymmetry between the two available time-directed descriptions", due to the lack of agential control and the violation of the Causal Markov Condition (CMC) (Farr 2020: 194).[6] As we can see, Farr mentions agential control as something that introduces a constraint on causal inference. If we accept (¬CTR), in cases involving everyday situations such as playing snooker, there are two causal variants, one of which is preferred. We give preference to the one in which a certain type of control is manifested. The privileged variant, which relies on "appeal to beliefs about the snooker player's agential control" (Farr 2020: 195), to determine a direction of causation, however, doesn't have to be the only option, as Farr has shown for such empirical cases. The acceptance of (¬CTR) seems to be 'an especially easy case' for mathematical explanations since we do not have distinct agential control in mathematics, so such privileged causal 'stories' are almost non-existent.[7] In other words, there is nothing strange in saying that A+B=C instead of C=A+B. It seems that this lack of privileged causal variant allows mathematical explanations, from the perspective of the epistemic agent, to be understood as compatible with the C-theory. Here, we easily affirm the acceptance of (¬CTR), since the distinction between cause and effect remains invariant in mathematical explanations. This means that our arguments have nothing to do with denying that causes temporally precede their effects, not even in the so-called epistemic domain. In other words, in embracing (¬CTR), the cause-effect relationship is unaffected.

At this point, we see that the C-theory is as sustainable as any other temporal theory, especially if we accept that there is nothing essential in the flow or direction of time. If we keep all of this in mind, and also take into account that we are considering strictly the domain of mathematics, the existence of symmetry in mathematical explanations may be compatible with the C-theory and time reversal.[8] The fact that we might encounter one form of symmetry in mathematics does not necessarily mean that we will find another form of it in that domain, but based on what we saw in the example of Zeitz's coin, mathematics acts as one of the only potential candidates for at least some form of temporal symmetry. Having this said, we will try to show that some examples of mathematical explanations do provide arguments for temporal bidirectionality.

4. Epistemic Temporal Symmetry in Mathematics

As we have underlined in previous sections, the lack of time direction in mathematical explanations, from the perspective of an epistemic agent, does not imply the lack of cause and effect in these kinds of explanations. Epistemic agent always understands explanations in a causal manner. But, the important difference between the B-theoretical (the A-theoretical approach is also implied here since the A-series also has a direction along with the flow) and the C-theoretical approach is that the C-theory allows time-directional symmetry. This means that cause and effect can be switched while retaining the causal asymmetry. That is to say that in a time direction D_1 an event a_1 can be the cause of the event b_1, and in time direction D_2, b_1 can be the cause of a_1. Explanatory causality remains asymmetric, just like in the B-theory, but its bidirectional nature allows that the same event can be both the cause and the effect in the two different temporal directions.

One other important thing to note here is that the previous section is not meant to imply that when we consider symmetry in mathematical explanations we automatically acknowledge that with the acceptance of the (¬CTR) we non-critically accept the existence of mathematical explanations which could be seen as epistemico-temporally bi-directional. Rather, the problem we wanted to address is the one which tackles the question could symmetry found in mathematical explanations be connected to time, since mathematical truths are generally considered to be lacking temporal (as well as spacial) relations. Having this in mind, we will now try to affirm that there is a temporal symmetry in mathematics (as mentioned, only within the 'epistemic application'). If we accept that in mathematical explanations, as is the case with Zeitz's coin, there are cases in which we have explanatory symmetry between the probability and the initial number of throws, we can rightfully conclude, as Lange did, that it requires an explanation, which indeed can be found. But could this symmetry be in any way also seen as epistemico-temporal? Let us start with an example related to Zeitz's coin, but much simpler. We know for a fact that where there is a certain number of coins thrown this implies a certain belief, but is it ever possible that the probability could imply the belief regarding the number of thrown coins? If we take into consideration something that we can label as *epistemic lack of directionality*, the answer could be affirmative. Almost anyone who has ever played the coin toss game is aware that the probability of getting a 50-50 heads/tails ratio increases with each subsequent toss. Throwing numbers certainly influence probability and it's a classic asymmetric math setup. But what would happen if we were to assume that the probability was affecting the agent's judgment about the number of throws? There is, strictly speaking, no reason to believe that the probability (if there was a way of knowing it in advance) would not affect the knowledge of the number of throws. If someone is, so to speak, not epistemicaly yet involved in the process, he could, in some cases, easily guess the number of throws by only knowing the probability.[9] To make our point clear, let us take a specific example in which we could compare the throw-to-probability ratio (which does not have a biased setting like Zeitz's coin). According to the widely accepted interpretation, probability comes as a time directed consequence of the analysis of the number of throws and the number of heads and tails. So if, for example, we were to throw a coin 50 times, and after those 50 throws the heads-to-tails ratio is 36-14 and we know that we will throw the coin 50 more times, as a consequence the probability will be in favor of reducing this difference. With that in mind, we conclude on probability based on the number of throws. That is, so to say, the easy part since it doesn't endanger our *epistemic bidirectionality* view.

Let us now take an example where, by some supernatural circumstances, we have the unmistakable insight that the probability of throwing a certain number of coins will be 46-54. Can we have any idea of the number of coins thrown on that basis? Obviously, based on this example, we can unequivocally know (since we are enabled by the *pre-epistemic bi-directionality* and the C-theory), for example, that a coin was not thrown less than 3 times, since the relationship between the two sides could not be expressed so percentage-wise. In addition, with some other mathematical calculations we can know approximately after how many throws this ratio could occur, and it is not difficult to imagine that there are other ways in which we could determine the potential number of throws. This is certainly not exact, but we must bear in mind that the inverse example does not project exactness either. That is since we are talking about knowledge, we are out of the realm of pure mathematics, and thus mathematical knowledge applied to the empirical world can have empirical inactivity. Hence, in this case, though it is undisputed that for a certain number of throws there is a certain precise probability, that probability says nothing about whether the number of throws will always coincide with its predictions. Likewise, the probability that is predicted in advance does not have to say anything about the number of throws. It seems that we do indeed encounter some kind of symmetrical temporal directionality here, though it could be classified as weak (in the sense that it does not talk about time reversal outside the given setting) and in the form of an epistemic implication. Nevertheless, this tells us that there are certain mathematical settings which indeed can, if we, conditionally, consider just their epistemic features[10] be treated as time symmetric.[11] Note that this applies only to those settings in which explanatory symmetry could be spotted. Even with that obstacle, explanatory symmetry cases seem sufficient for our point, since the idea is just to prove that the C-theory is compatible with epistemic processes concerning mathematics.

The simple example with the coin exemplified here is certainly not the only case of this potential symmetry. The same, even in a way for which could be said that is much easier, can be proven with Zeitz's coin, where it is absolutely irrelevant whether the number of throws 'comes before' probability or the probability 'comes before' the number of throws. When we say this, we naturally have in mind the fact that this affects the only the epistemic notion of temporal directionally. So, if the number of throws is, say, 50, we get a probability of 50/1 +1, on the other hand, if by any chance we know that the probability is 50/1 +1, we will also know that the number of throws is 50. Assuming all the aforementioned factors, we can conclude that the conditions of the example with Zeitz's coin, in addition to the symmetry in mathematical explanations discussed by Lange, and the short argumentation that we offered in this paper suggest that some mathematical settings provide a solid case for the existence of a symmetrical temporal directionality of epistemic processes related to mathematical knowledge.

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Notes

[1] Named after mathematician Paul Zeitz.
[2] For the original setting of the problem see Zeitz (2000).
[3] Since 1000 heads of 2000 throws represent a mean between 0 and 1, that is, the probability can be denoted as 0.500.
[4] We won't get into the details of the proof here, since they are thoroughly presented in both (Zeitz 2000) and (Lange 2017).
[5] There are many variations of both theories, which differ from each other to a greater or lesser extent.
[6] CMC can be defined as follows: "Let *G* be a causal graph with vertex set *V* and *P* be a probability distribution over the vertices in *V* generated by the causal structure represented by *G*. *G* and *P* satisfy the Causal Markov Condition if and only if for every *X* in *V*, *X* is independent of *V* (Descendants(*X*) ∪ Parents(*X*)) given Parents(*X*)" (Hausman and Woodward 1999: 522).
[7] This is not to say that the epistemic agent has no control over how he will approach a certain mathematical setting, but only that some elements of the mathematical explanation may occur to him without his intention.
[8] By this, we definitely do not mean pure mathematical domain, since if we were to assume one, it most certainly would not incorporate any kind of cause-effect relation and temporal directionality.
[9] This position could be considered pre-epistemic, since we are talking about agent's knowledge before tossing a coin, but it is not impossible to imagine that the toss has already taken place and that someone has told the agent the probability.
[10] This means that the symmetry holds as long as "we don't" start analyzing some mathematical proof "from one way or the other", in other words as long as the epistemic process does not "take up its direction".
[11] These settings are time symmetrical only for epistemic agents. As we have already noted, it would be far-fetched to claim that there is any temporality in the pure mathematical domain.